SSLC CLASS NOTES -Chapter 3

REAL

NUMBERS

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REAL NUMBERS

- **EUCLID'S DIVISION LEMMA**

$$a = (b \times q) + r \qquad 0 \leq r < b$$

a – Dividend ; **q** – quotient; **b** - Divisor; **r** – remainder

- Finding HCF of two positive integers Using this lemma.

If the divisor is a factor of the dividend, the last remainder will be zero.The last but one non-zero remainder willl be the H.C.F.

- **Prime numbers**

A positive integer 'p' is considered a prime number, if,
(i) p > 1 and
- **p dose not have factors other than 1 and p**

- **Composite numbers:**

A number greater than 1 and not a prime number is a composite number

- **Co-Primes:**

Two numbers 'a' and 'b' are said to be co-prime if the only common divisor of a' and b' is 1

ILLUSTRATIVE EXAMPLES

Example1: Find the largest number that divides 455 and 42 with the help of division algorithm.

42	455	10	← quotient
	420		
	35		← remainder

∴ 455 = (42x10) + 35

35	42	1	← quotient
	35		
	7		← remainder

∴ 42 = (35x1) + 7

7	35	5	← quotient
	35		
	0		← remainder

∴ 35 = (7x5) + 0
∴ HCF (455, 42) = HCF (42, 35) = HCF (35, 7) = 7
∴7 is the largest number that divides 455 and 42
Example2: : Show that every positive even integer is of the form 2q and every positive odd integer is of the form 2q + 1, where q is a whole number..
Sol: (i) Let 'a' be an even positive integer,
Apply division algorithm with a and b, where b = 2
$a = (2 \times q) + r$ where 0 □ r < 2
a = 2q + r where r = 0 or r = 1
since 'a' is an even positive integer, 2 divides 'a'
r = 0 ಆಗಿರಬೇಕು
∴ a = 2q + 0 ⇒ a = 2q
Hence, a = 2q when 'a' is an even positive integer
(ii) Let 'a' be an odd positive integer.
apply division algorithm with a and b, where b = 2
a = $(2 \times q) + r$ where 0 □ r < 2
a = 2q + r where r = 0 or 1
Here, $r \neq 0$ (∵a is not even) ⇒ r = 1
A = 2q + 1
Hence, a = 2q + 1 when 'a' is an odd positive integer.
$r \neq o \Rightarrow r = 1$
∴ a = 2q + 1
Example3: Use Euclid's division lemma to show that cube of any positive integer is either of the form 9m, 9m + 1 or 9m + 8 for some integer 'm'.
Sol: Let a and b be two positive integers, and a > b
∴ By Uclid's division lemma: a = bq + r ಇಲ್ಲಿ $0 \leq r < b$
Let b = 3,[multiply 9 by 3 we get cube number]
∴ a = 3q + r where $0 \leq r < 3$
(i) If r = 0 then a = 3q
$a^3 = (3q)^3 = 27q^3 = 9(3q^3) = 9m$ where $m = 3q^3$ and 'm' is an integer
(ii) If r = 1 then a = 3q + 1
$a^3 = (3q + 1)^3 = 27q^3 + 27q^2 + 9q + 1$
$= 9(3q^3 + 3q^2 + q) + 1$
= 9m+1,where $m = 3q^3+3q^2+q$ and 'm' is an integer
(iii) if r = 2 then a = 3q + 2
$a^3 = (3q + 2)^3 = 27q^3 + 54q^2 + 36q + 8$
$= 9(3q^2+6q^2+4q) +8 = 9m+8$ wherem $= 3q^2+6q^2+4q$ and 'm' is an integer

∴ cube of any positive integer is either of the form 9m, 9m+1 or 9m+8 for some integer m.

Example4: Prove that, if x and y are odd positive integers, then $x^2 + y^2$ is even but not divisible by 4.

Sol: We know that any odd positive integer is of the form 2q + 1, where q is an integer.

So, let x = 2m + 1 and y = 2n + 1, for some integers m and n .

∴ x = 2m + 1 and y = 2n = 1 [m,n]

$x^2 + y^2 = (2m + 1)^2 + (2n + 1)^2$

$x^2 + y^2 = 4m^2 + 4m + 1 + 4n^2 + 4n + 1$

$x^2 + y^2 = 4(m^2 + n^2) + 4(m + n) + 2$

$x^2 + y^2 = 4[(m^2 + n^2) + (m + n)] + 2$

$x^2 + y^2 = 4q + 2$, ಇಲ್ಲಿ $q = (m^2 + n^2) + (m + n)$

∴ $x^2 + y^2$ is even and leaves remainder 2 when divided by 4.

∴ $x^2 + y^2$ is even but not divisible by 4.

Example 5: A book seller has 28 Kannada and 72 English books. The books are of the same size. These books are to be packed in separate bundles and each bundle must contain the same number of books. Find the least number of bundles which can be made and also the number of books in each bundle.

Sol: To find the number of books in pack We have to find the HCF of 28 and 72 by Euclid's lemma

72 = 28x2 + 16

28 = 16x 1 + 12

16 = 12x 1 + 4

12 = 4X3 + 0

∴ H.C.F = 4

∴ Each bundle contains 4 books.

∴ No. of bundles of Kannada books = 28/4 =7

∴ No. of bundles of Engliss books = 72/4 = 18

Exercise 3.1

1. Use Euclid's division algorithm to find the HCF of the following numbers
   (i) 65 and 117 (ii) 237 and 81 (iii) 55 and 210 (iv) 305 and 793.

   (i) 65 and 117

   By Euclid's division lemma **a = bq + r**

65	117	1
	65	
	52	

By Euclids division algorithm
117 = 65x1 + 52

52	65	1
	52	
	13	

By Euclids division algorithm

65 = 52x1 + 13

13	52	4
	52	
	00	

By Euclids division algorithm
52 = 13x4 + 0

∴ **H.C.F.65 ಮತ್ತು 117 = 13**

(ii) 237 and 81

81	237	2
	162	
	75	

By Euclids division algorithm
237 = 81x2 + 75

75	81	1
	75	
	06	

By Euclids division algorithm
81 = 75x1 + 6

6	75	12
	72	
	03	

By Euclids division algorithm
75 = 6x12 + 3

03	06	12
	72	
	03	

By Euclids division algorithm
6 = 3x2 + 0

∴ The H.C.F of 237 and 81 = 3

(iii) 55 ಮತ್ತು 210

55	210	3
	165	
	45	

By Euclids division algorithm
210 = 55x3 + 45

45	55	1
	45	
	10	

By Euclids division algorithm
55 = 45x1 + 10

10	45	4
	40	
	05	

By Euclids division algorithm
45 = 10x4 + 5

05	10	2
	10	
	00	

By Euclids division algorithm
10 = 5x2 + 0

∴ The H.C.F of 55 and 210 = 5

(iv) 305 ಮತ್ತು 793

305	793	2
	610	
	183	
183	305	1
	183	
	122	
122	183	1
	122	
	61	
61	122	2
	122	
	00	

By Euclids division algorithm
793 = 305x2 + 183

By Euclids division algorithm
305 = 183x1 + 122

By Euclids division algorithm
183 = 122x1 + 61

By Euclids division algorithm
10 = 5x2 + 0

∴ The H.C.F of 305 and 793 = 61

1. Show that any positive even integer is of the form 4q or 4q + 2, where q is a whole number.

a' and b are positive integer and a > b.
By Euclids division algorithm,
a = bq + r ; $0 \leq r < b$
If b = 4,
a = (4x2) + r, $0 \leq r < 4$ ∴ r = 0, 1, 2, 3.
i) If r = 0,
a = 4q ⇒a = 2(2q) This is divided by 2 ∴ This is even
ii) If r = 1,
a = 4q + 1 ⇒ a = 2(2q) + 1 This is not divided by 2 ∴ This is odd.
iii) If r = 2,
a = 4q + 2 ⇒ a = 2(2q + 1) This is divided by 2 ∴ This is even.
iv) If r = 3,
a = 4q + 3 ⇒ a = 2(2q+1) + 1 This is not divided by 2 ∴ This is odd.
∴Any positive integer is of the form 4q and 4q+2

2. Use Euclid's division lemma to show that the square of any positive integer is either of the form 3m or 3m + 1 for some integer m, but not of the form 3m+2.

If any positive integer divided by 3, then the reminders are 0,1 and 2.
∴ a is of the form 3q, 3q + 1 or 3q + 2.
i) If a = 3q then, $a^2 = (3q)^2 = 9q^2 = 3(3q^2) = 3m$ ($m = 3q^2$)
ii) If a = 3q + 1 then, $a^2 = (3q + 1)^2 = 9q^2 + 6q + 1$
$= 3(3q^2 + 2) + 1 = 3m + 1$ ($m = 3q^2 + 2$)
iii) If a = 3q + 2 then, $a^2 = (3q + 2)^2 = 9q^2 + 12q + 4$
$\Rightarrow a^2 = 9q^2 + 12q + 3 + 1$
$\Rightarrow 3(3q^2 + 4q + 1) + 1 = 3m + 1$ $(m = 3q^2 + 4q + 1)$

From (i), (ii)and (iii)

∴ we can conclude that the square of any positive integer is either of the form 3m or 3m + 1 for some integer m, but not of the form 3m+2.

3. Prove that the product of three consecutive positive integers is divisible by 6.

Let the three consecutive integers are : n, n+1 and n + 2

By Euclids division algorithm,

$a = 6q + r ; 0 \leq r < 6$ (r = 0,1,2,3,4,5)

i) If r = 0 then, n = 6q

$\Rightarrow a = n(n+1)(n+2)$

$a = 6q (6q +1)(6q +2)$

$a = 6[q (6q +1)(6q +2)]$

This is divided by 6.

ii) If r = 1 then, n = 6q + 1

$\Rightarrow a = 6q + 1(6q + 1 +1)(6q + 1 +2)$

$a = (6q+ 1) (6q +2)(6q +3)$

$a = (6q+ 1) 2(3q +1)3(2q +1)$

$a = 6[(6q+ 1) (3q +1)(2q +1)]$

This is divided by 6.

iii) If r = 2 then, n = 6q + 2

$\Rightarrow a = 6q + 2(6q + 2 +1)(6q + 2 +2)$

$a = (6q+ 2) (6q +3)(6q +4)$

$a = 2(3q+ 1)3 (2q +1)(6q +4)$

$a = 6[(3q+ 1)(2q +1)(6q +4)]$

$a = 6(6q+ 1) (3q +1)(2q +1)$

This is divided by 6.

iii) If r = 3 then, n = 6q + 3

$\Rightarrow a = (6q + 3)(6q + 3 +1)(6q + 3 +2)$

$a = (6q+ 3) (6q +4)(6q +5)$

$a = 3(2q+ 1)2 (3q +2)(6q +4)$

$a = 6[(2q+ 1)(3q +2)(6q +4)]$

This is divided by 6.

iv) If r = 4 then, n = 6q + 4

$\Rightarrow a = (6q + 4)(6q + 4 +1)(6q + 4 +2)$

$a = (6q+ 4) (6q +5)(6q +6)$

$a = (6q+ 4) (6q +5)6(q +1)$

$a = 6[(6q+ 4) (6q +5)(q +1)]$

This is divided by 6.

v) r = 4 ಆದಾಗ, n = 6q + 5

ಆಗ $a = (6q + 5)(6q + 5 +1)(6q + 5 +2)$

$a = (6q+ 5) (6q +6)(6q +7)$

$a = (6q+ 5)6(q +1)(6q +7)$

$a = 6[(6q+ 5)(q +1)(6q +7)]$

This is divided by 6.

∴ The product of three consecutive positive integers is divisible by 6

4. There are 75 roses and 45 lily flowers. These are to be made into bouquets containing both the flowers. All the bouquets should contain the same number of flowers. Find the number of bouquets that can be formed and the number of flowers in them.

45	75	1
	45	
	30	

30	45	1
	30	
	15	

15	30	2
	30	
	00	

∴The H.C.F. of 75 and 45 = 15
∴ 15 of bouquets can be formed .
Each bouquet contains 75 ÷ 15 = 5 Rose and 45 ÷15 = 3 Lily.

5. The length and breadth of a rectangular field is 110m and 30m respectively. Calculate the length of the longest rod which can measure the length and breadth of the field exactly.

6.

30	110	3
	90	
	20	
20	30	1
	20	
	10	

7.

10	20	2
	20	
	00	

∴ The HCF. of 110 and 30 = 10
∴ length of the longest rod which can measure the length and breadth of the field exactly = 10ಮೀ

ILLUSTRATIVE EXAMPLES

Example1: Find the HCF and LCM of 18 and 45 by prime factorisation method.

Sol:18 = 2 x 3 x 3
45 = 3 x 3 x 5
∴ H.C.F. = 3 x 3 = 9 and
LCM = 2 x 3 x 3 x 5 = 90

Example2: Find the HCF and LCM of 42 and 72 by prime factorisation method i.e., by fundamental theorem of arithmetic.

Sol:42 = **2** x 3 x 7
72 = **2** x 2 x 2 x 3 x 3
∴ HCF = 2 and
LCM. = 2 x 2 x 2 x 3 x 3 x 7 = 3,024

Example3: Find the HCF of 344 and 60 by prime factorisation method. Hence find their LCM.

Sol:344 = 2 x 2 x 2 x 43
60 = 2 x 2 x 3 x 5
HCF(344,60) = 4
a x b = HCF(a,b) x LCM.(a,b)
344 x 60 = 4 x LCM.(a,b)
LCM(a,b) = $\frac{344 \ x \ 60}{4}$
LCM.(a,b) = 5160

Example4: Find the largest positive integer that will divide 150, 187 and 203 leaving remainders 6, 7 and 11 respectively.

Sol:Let the number be 'x'
150 is dividing by 'x" the remainder will be – 6
∴ 150 – 6 = 144 is exactly divisible by 'x'.
187 is dividing by 'x' the remainder will be – 7
∴ 187 – 7= 180 is exactly divisible by 'x'.
203 is dividing by 'x" the remainder will be – 11
∴ 203 – 11= 192 is exactly divisible by 'x'.
∴ 'x" is the H.C.F. of 144, 180 and 192
144 = 2 x 2 x 2 x 2 x 3 x 3
180 = 2 x 2 x 3 x 3 x 5
192 = 2 x 2 x 2 x 2 x 2 x 2 x 3
∴ x = 2 x 2 x 3 = 12

Example5: Find the smallest number that, when divided by 35, 56 and 91 leaves remainders of 7 in each case.

If the number is divisible by 35, 56 and 91 then it is the LCM of these numbers.
35 = 5 x 7
56 = 2 x 2 x 2 x 7
91 = 7 x 13
∴ LCM = 2 x 2 x 2 x 5 x 7 x 13 = 3,640
Since it leaves a remainder 7, the required number is 7 = 3,640 + 7 = 3,647

Example6: There is a circular path around a sports field. Sheela takes 36 minutes to drive one round of the field while Geeta takes 32 minutes to do the same. If they both start at the same point and at the same time and go in the same direction, after how many minutes will they meet again at the starting point.

Sol:: To find this, we have to find the LCM of 36 and 32
36 = 2 x 2 x 3 x 3
32 = 2 x 2 x 2 x 2 x 2
∴LCM. = 2^5 x 3^2 = 32 x 9 = 288
∴Sheela and Geeta meet again at the starting point after 288 minutes.

Exercise 1.2

1. Express each number as a product of prime factors.

(i) 120

2	120
2	60
2	30
3	15
5	5
	1

120 = 2x2x2x2x3x5

(ii) 3825

3	3825
3	1275
5	425
5	85
5	17
17	1

3825= 3x3x5x5x5

(iii) 6762

2	6762
3	3381
7	1127
7	161
23	23
	1

6762 = 2x3x7x7x23

(iv) 32844

2	32844
2	16422
3	8211
7	2737
17	391
23	23
	1

32844 = 2x2x3x7x17x23

2. If $25025 = P_1^{x_1}.P_2^{x_2}.P_3^{x_3}.P_4^{x_4}$. find the value of P_1, P_2, P_3, P_4, and x_1, x_2, x_3, x_4.

5	25025
5	5005
7	1001
11	143
13	13
	1

$25025 = 5^2 \times 7 \times 11 \times 13$

$\therefore P_1 = 5, P_2 = 7, P_3 = 11, P_4 = 13,$

$x_1 = 2, x_2 = 1, x_3 = 1, x_4 = 1$

3. Find the LCM and HCF of the following integers by expressing them as product of primes.

(i) 12,15 ಮತ್ತು 30

$12 = 2 \times 2 \times 3 = 2^2 \times 3$
$15 = 3 \times 5$
$30 = 2 \times 3 \times 5$
HCF = 3 and LCM = $2^2 \times 3 \times 5 = 60$

(ii) 18,81 and 108

$18 = 2 \times 3 \times 3 = 2 \times 3^2$
$81 = 3 \times 3 \times 3 \times 3 = 3^4$
$108 = 2 \times 2 \times 3 \times 3 \times 3 = 2^2 \times 3^3$
HCF = $3^2 = 9$ and LCM = $2^2 \times 3^4 = 4 \times 81 = 324$

4. Find the HCF and LCM of the pairs of integers and verify that LCM (a, b) × HCF (a, b) = a × b

(i) 16 and 80

$16 = 2 \times 2 \times 2 \times 2 = 2^4$
$80 = 2 \times 2 \times 2 \times 2 \times 5 = 2^4 \times 5$
HCF = 2^4 = 16 and LCM = $2^4 \times 5$ = 80
HCF(a,b) x LCM(a,b) = 16 x 80 = 1280
a x b = 16 x 80 = 1280
∴ LCM(a,b).x HCF(a,b). = a x b

(ii) 125 ಮತ್ತು 55

$125 = 5 \times 5 \times 5 = 5^3$
$55 = 5 \times 11$
HCF = 5 and LCM = $5^3 \times 11$ = 1375
LCM(a,b).x HCF(a,b) = 5 x 1375 = 6875
a x b = 125 x 55 = 6875
∴ LCM(a,b).x HCF(a,b). = a x b

5. If HCF of 52 and 182 is 26, find their LCM.

LCM(a,b).x HCF(a,b). = a x b
LCM(a,b).x 26= 52 x 182
LCM(a,b) $= \frac{52 \times 182}{26}$
LCM(a,b) = 2 x 182
LCM(a,b) = 364

6. Find the HCF of 105 and 1515 by prime factorisation method and hence find its LCM.

105 and 1515

$105 = 3 \times 5 \times 7$
$1515 = 3 \times 5 \times 101$
HCF = 15 and LCM = $5^3 \times 11$ = 10,605

7. Find the smallest number which when increased by 17 is exactly divisible by both 520 and 468.

$520 = 2 \times 2 \times 2 \times 5 \times 13 = 2^3 \times 5 \times 13$

$468 = 2 \times 2 \times 3 \times 3 \times 13 = 2^2 \times 3^2 \times 13$
$\therefore$ HCF $= 2^2 \times 13 = 4 \times 13 = 52$
$\therefore$ LCM $= 2^3 \times 3^2 \times 5 \times 13 = 8 \times 9 \times 5 \times 13 = 4680$
$\therefore 4680 - 17 = 4663$
The number 4663 when increased by 17is exactly divisible by 520 and 468.

8. A rectangular hall is 18m 72cm long and 13m 20cm broad. It is to be paved with square tiles of the same size. Find the least possible number of such tiles.
18m 72cm = 1872cm; 13m 20cm. = 1320cm
The area of the Hall = 1872 x 1320 sq.cm
$1872 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 13 = 2^4 \times 3^2 \times 13$
$1320 = 2 \times 2 \times 2 \times 3 \times 5 \times 11 \quad = 2^3 \times 3 \times 5 \times 11$
$\therefore$ HCF $= 2^3 \times 3 = 24$
$\therefore$ 1 Area of tiles = 24 x 24sq.cm
$\therefore$ Number of tiles required $= \frac{1872 \times 1320}{24 \times 24} = 4290$

9. In a school, the strength in 8^{th}, 9^{th} and 10^{th} standards are respectively 48, 42 and 60.Find the least number of books required to be distributed equally among the students of 8^{th}, 9^{th} and 10^{th}.
$48 = 2 \times 2 \times 2 \times 2 \times 3 = 2^4 \times 3$
$42 = 2 \times 3 \times 7$
$60 = 2 \times 2 \times 3 \times 5$
$\therefore$ LCM $= 2^4 \times 3 \times 5 \times 7 = 1680$
$\therefore$ The least number of books required to be distributed equally among the students =1680

10. x, y and z start at the same time in the same direction to run around a circular stadium. x completes a round in 126 seconds, y in 154 seconds and z in 231 seconds, all starting at the same point. After what time will they meet again at the starting point? How many rounds would have x, y and z completed by this time?
$126 = 2 \times 3 \times 3 \times 7 = 2 \times 3^2 \times 7$
$154 = 2 \times 7 \times 11$
$231 = 3 \times 7 \times 11$
$\therefore$ LCM $= 2 \times 3^2 \times 7 \times 11 = 1386$
$\therefore$ After 1386sce time will they meet again at the starting point.
By this time X completed $= \frac{1386}{126} = 11$ rounds
Y compleeted $= \frac{1386}{154} = 9$
Z compleeted $= \frac{1386}{231} = 6$

ILLSTRATIVE EXAMPLE

Example1:Prove that $5 - \sqrt{3}$ is an irrational number.
Proof: Let us suppose, $5 - \sqrt{3}$ is a rational number
$\Rightarrow 5 - \sqrt{3} = \frac{p}{q}$ [$q \neq 0$, and $(p,q) = 1$]
$\Rightarrow 5 - \frac{p}{q} = \sqrt{3}$
$\Rightarrow \frac{5q-p}{q} = \sqrt{3}$

$\frac{5q-p}{q}$ is a rational number and $\sqrt{3}$ is not a rational number
This gives us a contradiction.
∴ our supposition that 5 - $\sqrt{3}$ is a rational number is wrong
∴5 - $\sqrt{3}$ is an irrational number.

Example2;Prove that $\sqrt{3}+\sqrt{2}$ is a rational number .

Proof:Let us suppose $\sqrt{3}+\sqrt{2}$ is a rational number

$\Rightarrow \sqrt{3}+\sqrt{2} = \frac{p}{q}$ [$q \neq 0$, and (p,q) = 1]

$\Rightarrow \sqrt{3} = \frac{p}{q} - \sqrt{2}$

$\Rightarrow 3 = \frac{p^2}{q^2} - 2 \times \frac{p}{q}\sqrt{2} + 2$

$\Rightarrow 2 \times \frac{p}{q}\sqrt{2} = \frac{p^2}{q^2} + 2 - 3$

$\Rightarrow 2 \times \frac{p}{q}\sqrt{2} = \frac{p^2}{q^2} - 1$

$\Rightarrow \sqrt{2} = \frac{q(p^2-q^2)}{2pq^2}$

$\frac{q(p^2-q^2)}{2pq^2}$ is a rational number and $\sqrt{2}$ is a rational number
This gives us a contradiction..
∴ our supposition that $\sqrt{3}+\sqrt{2}$ is a rational number is wrong
$\sqrt{3}+\sqrt{2}$ is an irrational number

Example 3: Prove thant $\sqrt{2}$ is an irrational number.

Let us assume that $\sqrt{2}$ is a rational number.

$\Rightarrow \sqrt{2} = \frac{p}{q}$ ($p, q \in Z; q \neq 0$, p and q are co − prime)

$\Rightarrow \sqrt{2}q = p$

$\Rightarrow (\sqrt{2}q)^2 = p^2$

$\Rightarrow 2q^2 = p^2$

$\Rightarrow$ 2 divides p^2.

$\Rightarrow$ 2 divides p .∴ p is even

Let p = 2k where 'k' is an integer

$\Rightarrow p^2 = (2k)^2$

$\Rightarrow 2q^2 = (2k)^2$

$\Rightarrow 2q^2 = 4k^2$

$\Rightarrow q^2 = 2k^2$

$\Rightarrow$ 2 divides q^2.

$\Rightarrow$ 2 divides q .∴ q an even

∴ both 'p' and 'q' are even
∴ 'p' and 'q' have common factor 2
This is contradictory to our assumption that 'p' and 'q' are co-prime.
$\Rightarrow$our assumption that $\sqrt{2}$ is a rational number is wrong

∴ **$\sqrt{2}$ is an irrational number**

Exercise 1.3

1. Prove that $\sqrt{5}$ is an irrational number.

Let us assume that $\sqrt{5}$ is a rational number.
$\Rightarrow \sqrt{5} = \frac{p}{q}$ (p, q ∈ Z;q ≠ 0, p and q are co − prime)
$\Rightarrow \sqrt{5}q = p$
$\Rightarrow (\sqrt{5}q)^2 = p^2$
$\Rightarrow 5q^2 = p^2$
$\Rightarrow$ 5 divides p^2.
$\Rightarrow$ 5 divides p.
Let p = 5k where ‘k’ is an integer
$\Rightarrow p^2 = (5k)^2$
$\Rightarrow 5q^2 = (5k)^2$
$\Rightarrow 5q^2 = 25k^2$
$\Rightarrow q^2 = 5k^2$
$\Rightarrow$ 5 divides q^2.
$\Rightarrow$ 5 divides q.
∴ p and q have common factor 5.
This is contradictory to our assumption that ‘p’ and ‘q’ are co-prime.
⇒our assumption that $\sqrt{5}$ is a rational number is wrong
∴ $\sqrt{5}$ is an irrational number

2. Prove that the following are irrational numbers.

(i) $2\sqrt{3}$

Let us assume that $2\sqrt{3}$ is a rational number.
∴ $2\sqrt{3} = \frac{p}{q}$ ಆಗಿರಲಿ (p, q ∈ z;q ≠ 0)
$\Rightarrow \sqrt{3} = \frac{p}{2q}$
$\Rightarrow \sqrt{3}$ is a rational number ∵ $\frac{p}{2q}$ is a rational number.
But $\sqrt{3}$ is not a rational number .
This gives us a contradiction.
⇒our assumption that $2\sqrt{3}$ is a rational number is wrong
∴ $2\sqrt{3}$ is an irrational number.

(ii) $\frac{\sqrt{7}}{4}$

Let us assume that $\frac{\sqrt{7}}{4}$ is a rational number.
∴ $\frac{\sqrt{7}}{4} = \frac{p}{q}$ ಆಗಿರಲಿ (p, q ∈ z;q ≠ 0)

$\Rightarrow \sqrt{7} = \frac{4p}{q}$
$\Rightarrow \sqrt{7}$ is rational number ∵ $\frac{4p}{q}$ is rational.
But $\sqrt{7}$ is not a rational number
This gives us a contradiction.

$\Rightarrow$our assumption that $\frac{\sqrt{7}}{4}$ is a rational number is wrong

$\therefore \frac{\sqrt{7}}{4}$ is an irrational number.

(iii) $3 + \sqrt{5}$

Let us suppose $3 + \sqrt{5}$ is a rational number

$\Rightarrow 3 + \sqrt{5} = \frac{p}{q}$ [$q \neq 0$, and $(p,q) = 1$]

$\Rightarrow \sqrt{5} = \frac{p}{q} - 3$

$\Rightarrow \sqrt{5} = \frac{p-3q}{q}$ $\Rightarrow \sqrt{5} = \frac{q(p^2 + 2q^2)}{2pq^2}$

$\sqrt{5}$ is a rational number $\because$ $\frac{p-3q}{q}$ is a rational number

But $\sqrt{5}$ is not a rational number

This gives us a contradiction.

$\Rightarrow$our assumption that $3 + \sqrt{5}$ is a rational number is wrong

$\therefore 3 + \sqrt{5}$ is an irrational number.

(iv) $\sqrt{2} + \sqrt{5}$

Let us suppose $\sqrt{2} + \sqrt{5}$ is a rational number

$\Rightarrow \sqrt{2} + \sqrt{5} = \frac{p}{q}$ [$q \neq 0$, and $(p,q) = 1$]

$\Rightarrow \sqrt{2} = \frac{p}{q} - \sqrt{5}$

$\Rightarrow 2 = \frac{p^2}{q^2} - 2 \times \frac{p}{q}\sqrt{5} + 5$

$\Rightarrow 2 \times \frac{p}{q}\sqrt{5} = \frac{p^2}{q^2} + 5 - 2$

$\Rightarrow 2 \times \frac{p}{q}\sqrt{5} = \frac{p^2}{q^2} + 3 \Rightarrow \sqrt{5} = \frac{q(p^2+3q^2)}{2pq^2}$

$\sqrt{5}$ is a rational number $\because$ $\frac{q(p^2+3q^2)}{2pq^2}$ is rational number

But $\sqrt{5}$ is not a rational number

This gives us a contradiction..

$\therefore$ our supposition that $\sqrt{2} + \sqrt{5}$ is a rational number is wrong

$\therefore \sqrt{2} + \sqrt{5}$ is an irrational number

(v) $2\sqrt{3} - 4$

Let us suppose $2\sqrt{3} - 4$ is a rational number

$\Rightarrow 2\sqrt{3} - 4 = \frac{p}{q}$ [$q \neq 0$, and $(p,q) = 1$]

$\Rightarrow 2\sqrt{3} = \frac{p}{q} + 4$

$\Rightarrow \sqrt{3} = \frac{p+4q}{2q}$

$\Rightarrow \sqrt{3}$ is a rational number $\because$ $\frac{p+4q}{2q}$ is a rational number.

But $\sqrt{3}$ is not a rational number

This gives us a contradiction..

$\therefore$ our supposition that $2\sqrt{3} - 4$ is a rational number is wrong

$\therefore 2\sqrt{3} - 4$ is an irrational number